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Forecast in foreign exchange markets

R. Baviera[1,a], M. Pasquini[2], M. Serva[2], D. Vergni[3], and A. Vulpiani[3]

[1] Departement Finance et Économie, Groupe HEC, 78351 Jouy-en-Josas Cedex, France
[2] Dipartimento di Matematica, Università dell'Aquila & I.N.F.M. Unità dell'Aquila, 67100 Coppito, L'Aquila, Italy
[3] Dipartimento di Fisica, Università di Roma "La Sapienza"& I.N.F.M. Unità di Roma 1, 00185 P.le A. Moro 2, Roma, Italy

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Abstract. We perform a statistical study of *weak* efficiency in Deutschemark/US dollar exchange rates using high frequency data. The presence of correlations in the returns sequence implies the possibility of a statistical forecast of market behavior. We show the existence of correlations and how information theory can be relevant in this context.

PACS. 89.65.Gh Economics, business, and financial markets – 65.40.Gr Entropy and other thermodynamical quantities

1 Introduction

A large amount of research suggests that prices are related with information and in particular it focuses on efficiency in financial markets. A market is inefficient if a speculator can make a profit out of information present in the market. Since the celebrated work of Fama [1] a big effort has been done to test empirically and to understand theoretically the efficiency of financial markets.

A market is said to be efficient if prices "fully reflect" all available information, *i.e.* such information is completely exploited in order to determine the price, after having taken into account the costs to use this information and a transient time, due to costs, to reach equilibrium. The idea is that the investor destroys information while using it and as a consequence he contributes to produce equilibrium.

In the last years long term correlations have been observed in financial markets. We shall not review in details the contributions to the field. We stress that long term return anomalies are usually revealed *via* test of efficiency in a *semi-strong* form, *i.e.* not only considering asset prices but also some other publicly known news. The interest is generally focused on market reactions to an event occurred a fixed lag before (three to five years typically) such as divested firms [2], mergers [3] or initial public offerings [4,5]. Recent research (see *e.g.* [6–11]) has pointed out the existence of long range correlations also in the *weak* form. However only low frequency data are considered and implications on efficiency are not completely understood.

In this paper we focus on efficiency in the *weak* form, *i.e.* we consider only the information coming from historical prices. We are interested on a time scale longer than the typical correlation returns time (few minutes) but lower than the characteristic time after which we do not have statistical relevance of the results (roughly a couple of weeks): in this sense we deal with *long term* return anomalies. Currency exchange seems to be the natural subject for an efficiency test. We expect that such markets are very efficient as a consequence of their large liquidity.

For these reasons we have decided to analyze a one year high frequency dataset of the Deutschemark/US dollar exchange, the most liquid market. Our data, made available by Olsen and Associated, contains all worldwide $1,472,241$ bid-ask Deutschemark/US dollar exchange rate quotes registered by the inter-bank Reuters network over the period October 1, 1992 to September 30, 1993.

One of the main problem in tick data analysis, is the irregular spacing of quotes. In this paper we consider *business* time, *i.e.* the time of the transaction given by its rank in the sequence of quotes. This seems to be a reasonable way to consider time in a worldwide time series, where time delays and lags of no transaction are often due to geographical reasons. With *business* time we eliminate most of the *seasonality* in the financial signal.

In this paper we test the independence hypothesis of returns and define and measure the *available* information. In Section 2 we check the independence with two different techniques. The first one, called structure functions analysis, shows whether it is possible to rescale properly the distribution functions at different lags [12]. The second one is a direct independence test. The independence of two random variables x, y implies that $f(x)$ and $g(y)$ are uncorrelated for every f and g. We check it for $f(\cdot) = g(\cdot) = |\cdot|^q$. We interpret these quantities as an estimate of the correlation between returns of given size.

We want to quantify the *available* information and discuss its financial relevance. In Section 3 we consider a

[a] e-mail: bavierar@hec.fr

speculator with a given resolution, *i.e.* he is concerned only about fluctuations at least of size Δ. This reminds the ϵ entropy introduced by Kolmogorov [13] in the context of information theory. A similar filter has been first introduced by Alexander [14,15]. To show the inefficiency of the market he proposed the following trading rule: if the return moves up of Δ, buy and hold until it goes down of Δ from a subsequent high, then sell and maintain the short position till the return rises again of Δ above a subsequent low. In a similiar way we show that some information is available in financial series and how this information is related to market efficiency.

In Section 4 we summarize and discuss the results.

2 Long term correlations

After the seminal work of Bachelier [16], it was widely believed that price variations follow an independent, zero mean, Gaussian process. The main implications of the "fundamental principle" of Bachelier are that the price variation is a martingale and it is an independent random process.

Bachelier considers the market a "fair game": a speculator cannot exploit previous information to make better predictions of forthcoming events. Information can come only from correlations and in absence of them from the shape of the probability distribution of returns.

For about sixty years this contribution was practically forgotten, and quantitative analysis on financial data started again with advent of computers.

Following [1], we shall call hereafter "random walk" the financial models where the returns

$$r_t \equiv \ln \frac{S_{t+1}}{S_t} \tag{1}$$

are independent variables. In this paper we define S_t as the average between bid and ask price. We do not want to enter here in a detailed analysis of the huge literature about "random walk" models. We just mention that, before the contribution of Mandelbrot [17], the return r_t was considered well approximated by an independent Gaussian process. Mandelbrot proposed that returns were distributed according a Levy-stable, still remaining independent random variables.

At present, it is commonly accepted that the variables

$$r_t^{(\tau)} \equiv \sum_{t'=t}^{t+\tau-1} r_{t'} = \ln \frac{S_{t+\tau}}{S_t} \tag{2}$$

do not behave according a Gaussian at small τ, while the Gaussian behavior is recovered for large τ. Of course a return r_t distributed according to a Levy, as suggested by Mandelbrot, is stable under composition and then also $r_t^{(\tau)}$ would follow the same distribution for every τ. A recent proposal is the truncated Levy distribution model introduced by Mantegna and Stanley [18] which fits well the data and reproduces the transition from small to large τ.

The other deviation from a behavior *à la* Bachelier comes from the presence of correlations in the financial signal. In this paper we focus our attention on independence tests. We remark once again that an influence of the return r_t at time t on the return $r_{t+\tau}$ at time $t+\tau$ implies a not fully efficient market in a *weak* form. The relevance of the question is clear in the case of an investor who analyzes historical data to forecast market behavior and to make a profit out of this information.

As a test of independence it is generally considered the correlation functions on time intervals τ

$$C(\tau) \equiv \langle r_t r_{t+\tau} \rangle - \langle r_t \rangle \langle r_{t+\tau} \rangle \ , \tag{3}$$

where $\langle \cdot \rangle$ denotes the temporal average

$$\langle A \rangle \equiv \frac{1}{T} \sum_{t=1}^{T} A_t$$

and T is the size of the sample.

The presence of correlations in Deutschemark/US dollar exchange returns before the nineties is a well known fact. For example [19], who consider the same dataset we use, show that the returns are negatively correlated for about three minutes. However the presence of long term memory (*e.g.* in a lag up to two weeks) and its consequences on investment rules have not been shown up to now.

A sort of long term memory can be revealed with appropriate tools, see for example the seminal works in the field [14,15] and [20], and the most recent literature [6–9,21,22], where it is shown that absolute returns or powers of returns exhibit a long range correlation. It is a common belief that it is not possible to exploit this kind of information because of transaction costs.

We shall show in the next section that dependent returns have a clear financial meaning, because they imply the existence of *available* information.

In Section 2.1 we show the persistence of a long range memory for the Deutschemark/US dollar exchange rate by means of the analysis of structure functions. In Section 2.2, we test directly the independence of returns with a generalization of the correlation analysis.

2.1 Structure functions

There is some evidence that the process $r_t^{(\tau)}$ cannot be described in terms of a unique scaling exponent [23,24], *i.e.* it is not possible to find a real number h such that the statistical properties of the new random variable $r_t^{(\tau)}/\tau^h$ do not depend on τ.

The scaling exponent h gives us information on the features of the underlying process. In the case of independent Gaussian behavior of r_t the scaling exponent is 1/2.

On the contrary, the data show that the probability distribution function of $r_t^{(\tau)}/\sqrt{\mathrm{Var}[r_t^{(\tau)}]}$ changes with τ [23,24]. This is an indication that r_t is a dependent

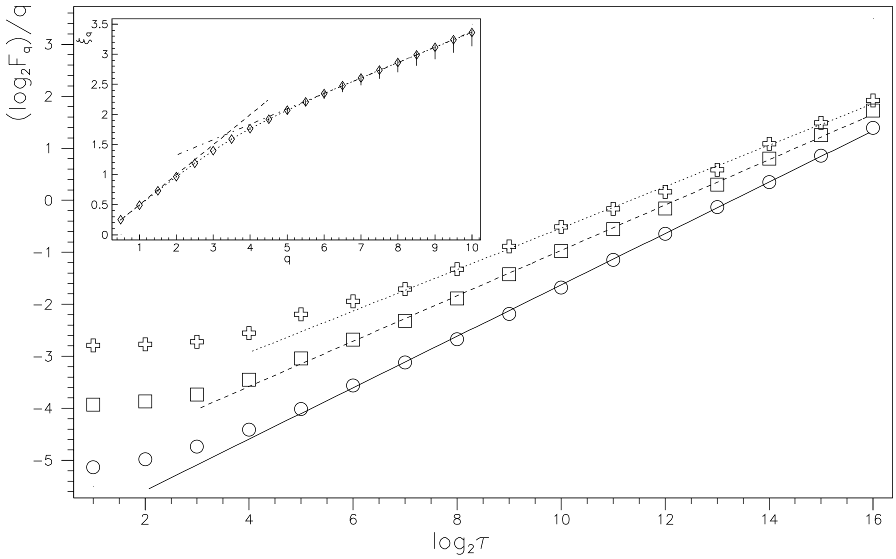

Fig. 1. Structure functions $\frac{1}{q}\log_2 F_q(\tau)$ *versus* $\log_2 \tau$ for Deutschemark/US dollar exchange rate quotes. The three plots correspond to different value of q: $q = 2.0$ (○), $q = 4.0$ (□) and $q = 6.0$ (+). In the inset we show ξ_q *versus* q. We estimate with linear regression two different regions in this graph. The first one is a line of slope 0.5 (dashed line), and the second has a slope 0.256 (dash dotted line).

stochastic process and it implies the presence of wild fluctuations.

A way to show these features, which is standard for the fully developed turbulence theory [25], is to study the structure functions:

$$F_q(\tau) \equiv \langle |r_t^{(\tau)}|^q \rangle \ . \tag{4}$$

In the simple case where r_t is an independent random process, one has (for a certain range of τ)

$$F_q(\tau) \sim \tau^{hq} \ , \tag{5}$$

where $h > 1/2$ in the Levy-stable case while the Gaussian behavior is recovered for $h = 1/2$. The truncated Levy distribution corresponds to $h > 1/2$ for τ sufficiently small and to $h = 1/2$ at large τ. "Random walk" models present always a unique scaling exponent. If the structure function has the behavior in (5) we call the process self-affine (sometimes called uni-fractal).

As previously mentioned a description in terms of a unique scaling exponent h does not work. Therefore instead of (5) one has

$$F_q(\tau) \sim \tau^{\xi_q} \ , \tag{6}$$

where ξ_q are called scaling exponents of order q. If ξ_q is not linear, the process is called multi-affine (sometimes multi-fractal). Using simple arguments it is possible to see that ξ_q has to be a convex function of q [26]. The larger is the difference of ξ_q from the linear behavior in q the wilder are the fluctuations and the correlations of returns. In this sense the deviation from a linear shape for ξ_q gives an indication of the relevance of correlations.

In Figure 1 we plot, the $F_q(\tau)$ for three different values of q. A multi-affine behavior is exhibited by different slopes of $\frac{1}{q}\log_2(F_q)$ *vs.* $\log_2(\tau)$, at least for τ between 2^4 and 2^{15} *business* times (which roughly correspond to few minutes and two weeks respectively). For larger lags a spurious behavior can arise because of the finite size of the dataset considered. In the inset we plot the ξ_q estimated by standard linear regression of $\log_2 F_q(\tau)$ *vs.* $\log_2(\tau)$ for the values of τ mentioned before. To give an estimation of errors, the most natural way turns out to be a division of the year dataset in two semesters. This is natural in the financial context, since it is a measure of reliability of the second semester forecast based on the first one. We observe that the traditional stock market theory (Brownian motion for returns), gives a reasonable agreement with $\xi_q \simeq q/2$ only for $q < 3$, while for $q > 6$ one as $\xi_q \simeq \tilde{h}q + c$ with $\tilde{h} = 0.256$ and $c = 0.811$. We stress once again that such a behavior cannot be explained by a "random walk" model (or other self-affine models) and this effect is a clear evidence of correlations present in the signal.

2.2 Long term correlations analysis

Let us consider the absolute returns series $\{|r_t|\}$, which is often shown to be long range correlated in recent literature [6–9,21,22]. Absolute values mean that we are interested only in the size of fluctuations.

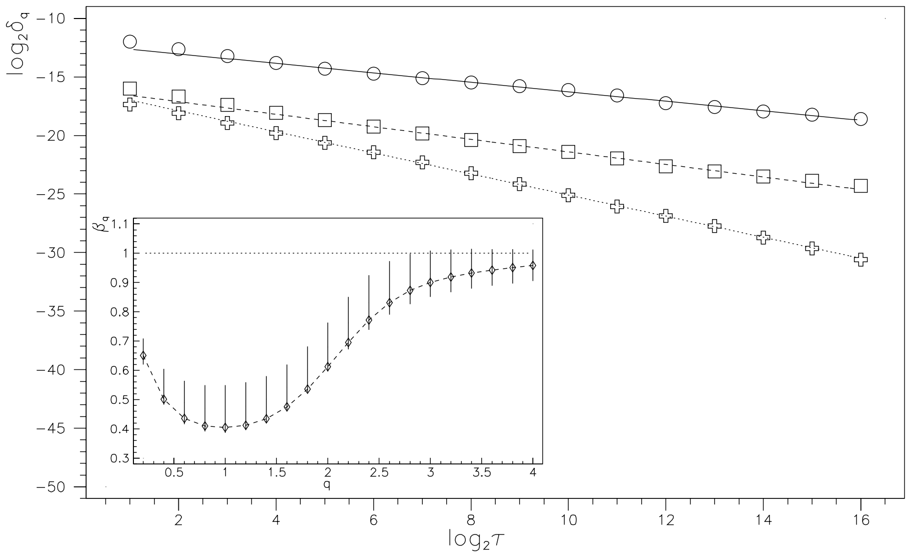

Fig. 2. $\log_2 \delta_q$ *versus* $\log_2 \tau$. The three plots correspond to different value of q: $q = 1.0$ (○), $q = 1.8$ (□) and $q = 3.0$ (+). In the inset we show β_q *versus* q, the horizontal line shows value $\beta = 1$ corresponding to independent variable.

Let us introduce the generalized correlations $C_q(\tau)$:

$$C_q(\tau) \equiv \langle |r_t|^q |r_{t+\tau}|^q \rangle - \langle |r_t|^q \rangle \langle |r_{t+\tau}|^q \rangle \ . \tag{7}$$

We shall see that the above functions will be a powerful tool to study correlations of returns with comparable size: small returns are more relevant at small q, while $C_q(\tau)$ is dominated by large returns at large q (the usual definition of correlation for absolute returns is recovered for $q = 1$).

Following the definitions in [27], let us suppose to have a long memory for the absolute returns series, *i.e.* the correlations $C_q(\tau)$ approaches zero very slowly at increasing τ, *i.e.* $C_q(\tau)$ is a power-law:

$$C_q(\tau) \sim \tau^{-\beta_q}.$$

Instead of directly computing correlations $C_q(\tau)$ of single returns we consider rescaled sums of returns. This is a well established way, if one is interested only in long term analysis, in order to drastically reduce statistical errors that can affect our quantities [28]. Let us introduce the *generalized cumulative absolute returns* [10]

$$\chi_{t,q}(\tau) \equiv \frac{1}{\tau} \sum_{i=0}^{\tau-1} |r_{t+i}|^q \tag{8}$$

and their variance

$$\delta_q(\tau) \equiv \langle \chi_{t,q}(\tau)^2 \rangle - \langle \chi_{t,q}(\tau) \rangle^2 \ . \tag{9}$$

After some algebra one can show that if $C_q(\tau)$ for large τ is a power-law with exponent β_q, then $\delta_q(\tau)$ is a power-law with the same exponent:

$$C_q(\tau) \sim \tau^{-\beta_q} \quad \Longrightarrow \quad \delta_q(\tau) \sim \tau^{-\beta_q} \quad \beta_q < 1 \ .$$

If $|r_t|^q$ is an uncorrelated process one has that $\delta_q(\tau)$ scales with $\beta_q = 1$.

In other words the hypothesis of long range memory for absolute returns ($\beta_q < 1$), can be checked *via* the numerical analysis of $\delta_q(\tau)$.

In Figure 2 we plot the δ_q *vs.* τ in log-log scale, for three different values of q. The variance $\delta_q(\tau)$ is affected by small statistical errors, and it confirms the persistence of a long range memory for a τ larger than 2^4 and up to 2^{15}, as in Section 2.1.

The exponent β_q can be profitably estimated by standard linear regression of $\log_2(\delta_q(\tau))$ *versus* $\log_2(\tau)$, and the errors are estimated in the same way of Section 2.1.

We notice in the inset that the "random walk" model behavior is remarkably different from the one observed in the Deutschemark/US dollar exchange for $q < 3$. This implies the presence of strong correlations, while one has $\beta_q = 1$ for large values of q, *i.e.* big fluctuations are practically independent.

3 Available information

Let us focus our attention on information analysis of the return r_t. We must treat the dataset in such a way that methods of information theory can be applied.

Let us remind some basic steps needed to use information theory in time series analysis:

– Obtain a discrete symbolic sequence from the original signal r_t:

$$c_1, c_2, \ldots, c_i, \ldots$$

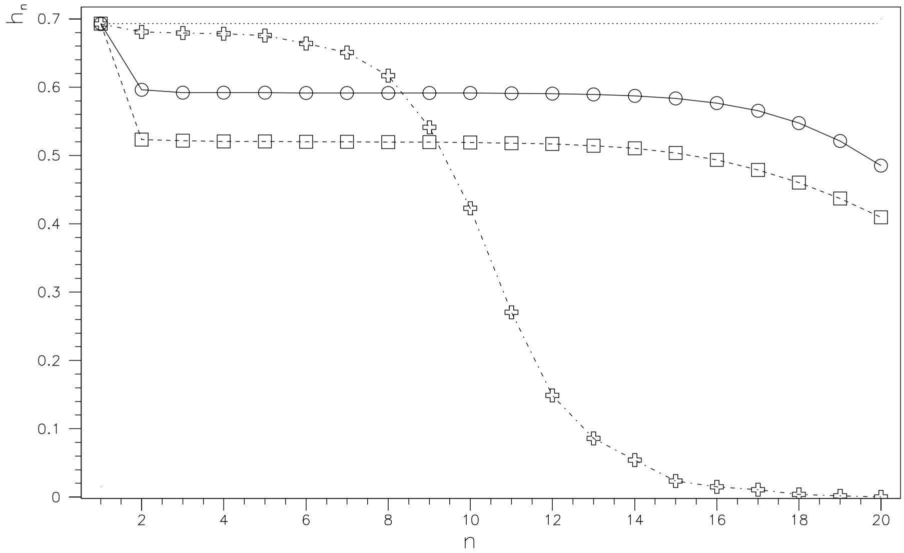

Fig. 3. h_n *versus* n. The three plots correspond to different value of Δ: $\Delta = 0.00005$ ($\circ$), $\Delta = 0.0002$ ($\Box$) and $\Delta = 0.004$ ($+$). The dotted line indicates $\ln(2)$.

where each c_i takes only a finite number, say m, of values.

- Consider a sequence of n symbols $C_n = \{c_1, c_2, \ldots, c_n\}$, its probability $p(C_n)$ and the block entropy H_n

$$H_n \equiv -\sum_{C_n} p(C_n) \ln p(C_n) \ . \qquad (10)$$

- The difference

$$h_n \equiv H_{n+1} - H_n \qquad (11)$$

represents the average information needed to specify the symbol c_{n+1} given the previous knowledge of the sequence $\{c_1, c_2, \ldots, c_n\}$.

The series of h_n is monotonically not increasing and for an *ergodic* process one has

$$h = \lim_{n \to \infty} h_n \qquad (12)$$

where h is the Shannon entropy [29].

It is easy to show that if the stochastic process $\{c_1, c_2, \ldots\}$ is Markovian of order k (*i.e.* the probability to have c_n at time n depends only on the previous k steps $n-1, n-2, \ldots, n-k$), then $h_n = h$ for $n \geq k$. The maximum value of h is $\ln(m)$. It occurs if the process has no memory at all and the m symbols have the same probability.

The difference between $\ln(m)$ and h is intuitively the quantity of information we may use to forecast future behavior, given the present information. We define *available* information:

$$I \equiv \ln(m) - h = R \ln(m) \qquad (13)$$

where $R = 1 - h/\ln(m)$ is called the *redundancy* of the process [29].

Hereafter we limit the discrete process to take only two values, -1 and 1 which have an evident financial meaning: the symbol -1 occurs if the stock price decreases, otherwise the symbol is 1.

The procedure to create the symbolic sequence is :

- we fix a resolution value Δ and we define

$$r_{t,t_0} \equiv \ln \frac{S_t}{S_{t_0}} \ , \qquad (14)$$

where t_0 is the initial *business* time, and $t > t_0$. We wait until an exit time t_1 such as:

$$|r_{t_1,t_0}| \geq \Delta \ .$$

In this way we consider only market fluctuations of amplitude Δ.

- following the previous prescription we create a sequence of returns

$$\{r_{t_1,t_0}, r_{t_2,t_1}, \ldots, r_{t_k,t_{k+1}}, \ldots\} \ ,$$

from which we obtain the symbolic dynamics:

$$c_k = \begin{cases} -1 \text{ } if \ r_{t_k,t_{k+1}} < 0 \\ +1 \text{ } if \ r_{t_k,t_{k+1}} > 0 \end{cases} \ . \qquad (15)$$

In Figure 3 it is shown that the entropy is clearly different from $\ln(2)$ in a wide range of Δ, *i.e.* there is a set of Δ for which the *available* information (see Eq. (13)) is very large.

In Figure 4 we plot the *available* information *versus* Δ and the distribution of transaction costs. Because these

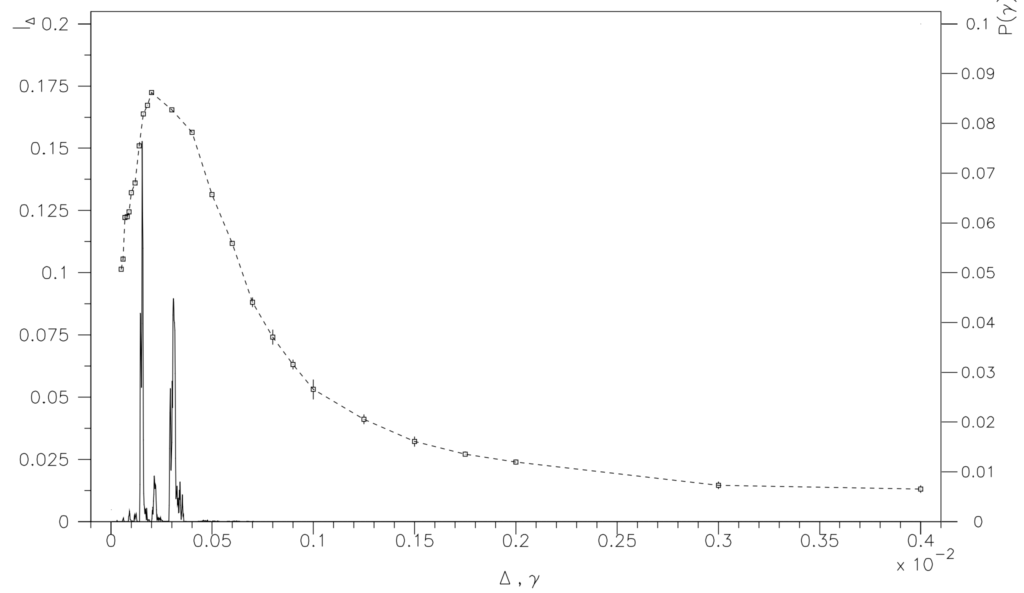

Fig. 4. *Available* information I_Δ *versus* Δ (on the left), superimposed to the distribution of transaction costs, $P(\gamma)$ *versus* γ (on the right).

two quantities do not have similar size they are plotted on different vertical scales but they are superimposed to make easier comparison between them. We observe that the maximum of the *available* information is almost in correspondence to the maximum of the distribution of the transaction cost.

We have estimated the transaction costs γ as

$$\gamma_t = \frac{1}{2} \ln \frac{S_t^{(\text{ask})}}{S_t^{(\text{bid})}} \simeq \frac{S_t^{(\text{ask})} - S_t^{(\text{bid})}}{2S_t^{(\text{bid})}} \cdot$$

It can be shown, following Kelly [30], that the *available* information is equal to the growth rate of capital following a given trading rule (and forgetting the costs involved).

It is then easy to comment the shape of the *available* information shown in Figure 4.

The speculator cannot have a resolution Δ lower than the transaction costs, profits from such an investment would be in fact less than costs. For Δ fluctuation larger than the typical transaction cost it starts to become possible to use part of the information present in the market and the *available* information decreases. For large Δ, all investors are able to discover the *available* information and to make it profitable with a feasible strategy. As a consequence, the efficient equilibrium is then restored for all practical purposes.

We want to stress, however, that once transaction costs are included and liquidity constraints are properly considered (the speculator should use heavily leverage to be able to use *available* information) the efficiency hypothesis is practically restored for all Δs.

4 Conclusions

In this paper we have considered the long term anomalies in the Deutschemark/US dollar quotes in the period from October 1, 1992 to September 30, 1993 and we have analyzed the consequences on the *weak* efficiency of this market.

In Section 2 we have shown the presence of long term anomalies with two techniques: the structure functions and a generalization of the usual correlation analysis. In particular we have pointed out that "random walk" models (or other self-affine models) cannot describe these features.

Once we have shown the existence of correlations in financial process, we have tested whether they allow for a profitable strategy.

With such a goal in mind, in Section 3 we have measured the *available* information with a technique which reminds the Kolmogorov ϵ entropy. An investor, who waits to modify his portfolio till the asset has a fluctuation Δ, observes a finite *available* information.

However, the existence of such a trading rule does not imply that the investment is feasible in practice. Namely it can be shown that when realistic investments are involved almost no *available* information survives.

The technique described here can be considered as a powerful tool to test *weak* efficiency and a graphical way to show how speculation contributes to reach efficient equilibria destroying the *available* information present in the market.

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